A Celebration of the Contributions of Art Cox to Stellar Pulsation Interpretations

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Abstract. A roughly chronological account is given of Arthur N. Cox's published work of 1953–1996 in, mostly, stellar pulsation theory, with a digression into stellar opacity. When possible, his work is placed in the context of the contemporary efforts.

1. Introduction

Fifty years ago, in 1947, Svein Rosseland (1964) wrote his seminal book *Pulsation Theory of Variable Stars*, which collected all the important early work on stellar pulsation theory. In that same year Art Cox first came to Los Alamos National Laboratory from Caltech, where he was an undergraduate, to work as a summer student. Both pulsation theory and Art have advanced considerably since then. Art's imprint on stellar pulsations has been quite sizable, and this short article will attempt to summarize it.

Art's biography appears elsewhere in this volume, but it may be noted here that after graduating from Caltech in 1948, Art entered the University of Indiana, where he became one of the first of the Astronomy Ph. D. recipients in 1954. Art immediately joined Los Alamos National Laboratory, where he applied his skills to experimental diagnostics of above-ground nuclear tests. In this work he shortly became the leader of the J-15 group. With the cessation of atmospheric testing in the 1960s, Art and J-15 began giving some of their attention to stellar evolution and pulsation, aided and abetted by a long list of colleagues from the University of Indiana: Robert Brownlee, John Cox, Paul Mutschlecner, David King and several others. John Cox in particular, who had done his Ph. D. thesis on the role of the He II ionization zone in driving cepheid pulsation, may have stimulated the work on cepheids. After a stint in the 1970s as a Program Director at the NSF, Art returned to the Theoretical Division of LANL, and entered a period of working prolifically on linear and non-linear pulsations of almost every kind of variable star known, as he continues to do today.

2. Indiana Years—Stellar Photometry

His thesis was on the subject of stellar photometry, and his first published paper (Cox 1953), in the Astrophysical Journal, was on the subject of transferring the photometric standards from the northern to the southern hemisphere. For his thesis he studied the color-magnitude diagram of the southern galactic cluster NGC 2287 (Cox 1954). This work was contemporary with that of Sandage and Hubble, followed by Arp, on several northern galactic and globular clusters, also using color-magnitude diagrams to forge

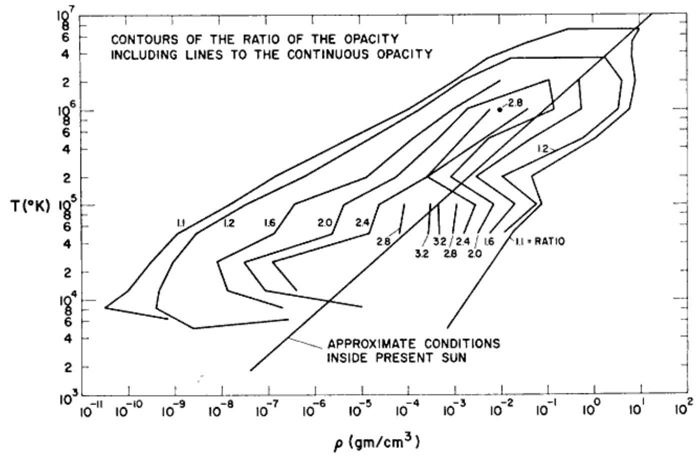

Figure 1. Contours of the ratio of total radiative opacity to continuous opacity for the Aller mixture. (Reproduced from *The Astrophysical Journal Supplement,* by permission.)

the observational basis for the theory of stellar evolution. It was observational stellar evolution's loss—but stellar pulsation's gain—when Art left this work aside to go to LANL.

3. Astrophysical Opacities

No discussion of Art Cox's career would be complete without mentioning opacities: the Rosseland mean mass absorption coefficient for radiation by matter. Early work on stellar structure, and also stellar atmospheres, had used the hydrogenic continuous absorption coefficients, as, *e.g.*, those of Keller and Meyerott (1955). In a series of papers (Cox 1964; Cox 1965; Cox, Stewart and Eilers 1965; Cox and Stewart 1965; Cox 1966) Art and his colleagues showed the considerable importance of including the bound-bound contribution to the opacity. This is demonstrated in Figure 1 from Cox, Stewart and Eilers (1965). These opacities, and their successors created by the efforts of Magee, Merts, and Huebner in addition to Stewart and Eilers, were a foundation stone of thirty years of stellar structure calculations.

The features of the relation between opacity and temperature play a major role in pulsation theory; this idea will recur below.

4. Numerical Stellar Hydrodynamics and the Cepheids

We turn the clock back to 1960; Art and Bob Brownlee presented one of the first numerical calculations of the evolution of the sun from the zero-age main sequence to its present state (Cox and Brownlee 1960, Brownlee and Cox 1961). The *Sky and Telescope* article is especially notable. It describes, *in words alone,* how a Lagrangian hydrodynamics calculation proceeds step-by-step in time. At the time of these articles a state-of-the-art scientific computer was an IBM 704, as shown in the photo of LANL's computer center in Figure 2.

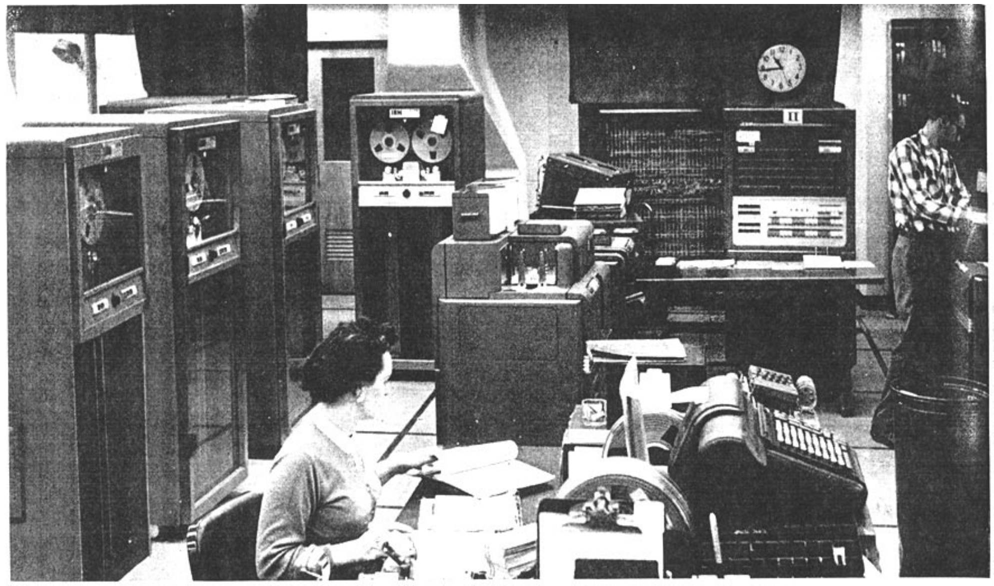

Figure 2. A view of the LANL computer room *circa* 1960, used for the first solar evolution calculations. The Marchant calculator used for checking the computer results is visible on the table. (From *Sky and Telescope* magazine, by permission.)

In the early 1960s the breakthrough work on numerical simulations of stellar pulsation began. Robert Christy studied the RR Lyrae stars (Christy 1962, 1964 and 1966) and, with the inspiration of John Cox, Art and his colleagues studied the cepheids (Cox and Olsen 1963; Cox, Cox and Olsen 1963; Cox, Brownlee and Eilers 1966; Cox, Cox, Olsen, King and Eilers 1966). The computer code described by Cox and Olsen is apparently the same one outlined in words by Brownlee and Cox, although hydrostatic equilibrium had been assumed for the sun, of course. The following AAS paper by both Coxes and Olsen gives the computational verification of John Cox's and Whitney's (1958) (and Zhevakin's 1953) model of overstability due to the He II ionization zone. The Cox, Brownlee and Eilers paper is the full description of the numerical technique used for the LANL pulsation calculations. Both this method and the one Christy described in his 1964 article grew out of the earlier experience with numerical hydrodynamics of people such as von Neumann, as passed down within the LANL community and as documented by Richtmyer (1957), among others.

The 1965 CCOKE paper demonstrated in the most convincing fashion that the oscillations of a cepheid envelope can be self-excited and do grow until a periodic limit-cycle oscillation is reached. This is shown in Figure 3. The growth remains exponential over many decades, so, although the models *can* be started from machine noise, a considerable amount of computer time is saved by starting from a larger amplitude that is still within the linear domain. The low growth rate of the instability, of order 2–3% per period, means that hundreds of periods still must be calculated, as shown. At the end of the calculation the oscillation is perfectly periodic, apart from the effects of numerical errors due to finite zoning and time steps. This is seen in Figure 4, also from CCOKE. The shapes of the light and velocity curves CCOKE found were

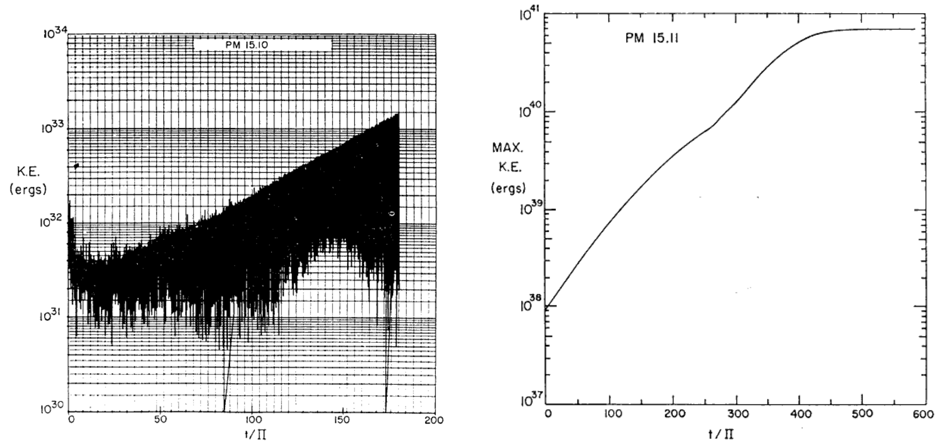

Figure 3. The growth in time of oscillations of one of the cepheid models from machine noise (on left) and an arbitrary higher amplitude (on right) is shown as computed by Cox, Cox, Olsen, King and Eilers. Abscissa: time measured in pulsation periods; ordinate: total kinetic energy in the stellar envelope. In the right panel only the maximum for each cycle is shown. (From *The Astrophysical Journal,* by permission.)

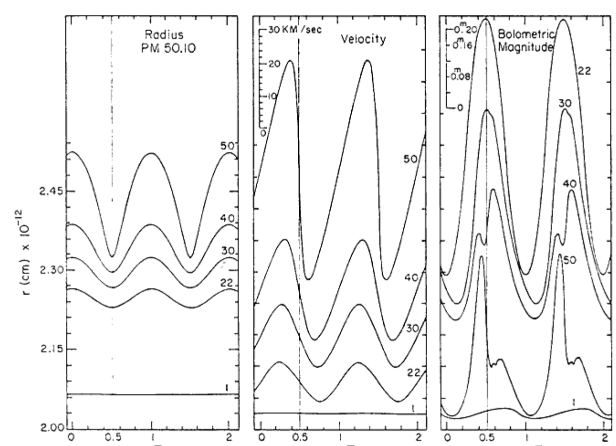

Figure 4. The limit-cycle variations of the radius, velocity and luminosity of selected mass shells for a cepheid model in Cox, Cox, Olsen, King and Eilers. Velocity is shown as positive away from the earth, *i.e.,* inward. (From *The Astrophysical Journal*, by permission.)

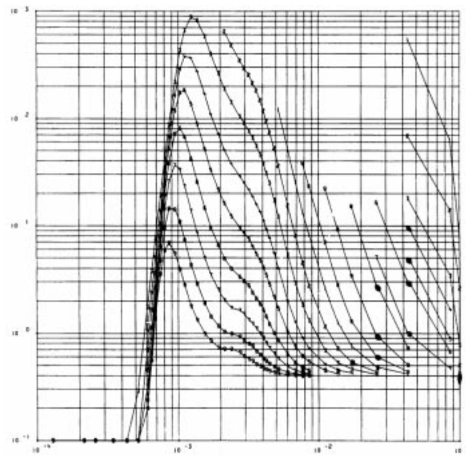

Figure 5. Rosseland mean opacity *vs.* temperature (in keV) at a number of densities for the Iben I mixture in Cox, King and Tabor (1973). (From *The Astrophysical Journal,* by permission.)

quite similar to observations, except that, since the model stellar envelope did not go all the way to the center, the mode that was found was similar to a first overtone for a full model. Also, the zoning was too coarse outward of the hydrogen ionization zone to give the proper phase relation between light and velocity. These things were corrected in later models.

A few years later Art and his group began combining linear nonadiabatic stability analysis with the nonlinear hydrodynamic calculations. Along the way a massive set of astrophysical opacities was computed by Cox and Tabor (1976). These included forty different astrophysical mixtures. The linear studies had shown how sensitive the pulsational stability was to small wiggles in the opacity *vs.* temperature in the He II ionization region, so many of the new tables had exquisite detail in that area, as seen in Figure 5.

Meanwhile, Art pursued another interest: solar eclipses. He organized a group of astronomers from LANL and elsewhere to observe several eclipses using a specially fitted KC-135 aircraft and other facilities. We see in Figure 6 (from S&T vols. 39 and 46) what it was like for Art to make one of these flights.

5. Anomalous Masses

From the work of Christy (1968) onward, it was known that the masses that had to be assumed for cepheids in order to get the pulsational properties right disagreed with the masses inferred from observed luminosities using stellar evolution theory. This appeared in the normalization constant in the period-luminosity relation, and also in the mass needed to cause a pronounced double peak in the light curve. But perhaps the most glaring disagreement was with the mass inferred from the two periods exhibited by the small class of double-mode cepheids (Petersen 1973). In this case the uncertainties of effective temperature, bolometric correction and distance did not enter, and the problem

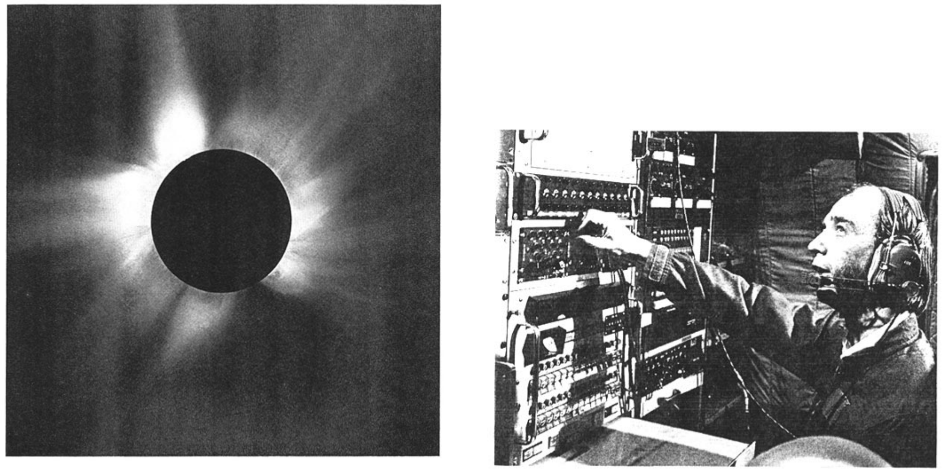

Figure 6. The left panel shows an image of the 1970 Mexico eclipse, which Art's team observed. The right panel shows Art leading the team on the NC-135 to observe the 1973 Africa eclipse. The headset connected Art to the nine separate stations on the aircraft, from which 32 people from 11 institutions were making observations. (From *Sky and Telescope* magazine, by permission.)

was unavoidable. For more than twenty years this remained the outstanding problem in stellar pulsation.

Art picked up the gauntlet in 1975, when King, Cox, Eilers and Cox (1975) did a non-linear study of the double-mode cepheid U TrA and confirmed the discrepancy. The mass problem continued to occupy Art's attention for about fifteen years. Cox, Hodson and King (1978) verified that the period ratio inferred from a non-linear calculation of a mixed-mode model agreed with the ratio of periods in the linear theory. Cox, Deupree, King and Hodson (1977) found that the altered density and pressure stratification due to a surface layer enhanced in helium did, in fact, correct the period ratios in the needed way. This idea of surface helium enhancement was pursued by Art and his colleagues for some time afterward (Cox, King and Hodson 1978; Cox, Michaud and Hodson 1978). The subject of the cepheid masses was reviewed by Art in 1978 and again in 1980 (Cox 1978,1980).

The RR Lyrae stars presented a somewhat different problem. The double-mode stars can be explained without invoking a surface He enhancement(Cox, King and Hodson 1980), but the inferred mass for RR Lyraes in Oosterhoff type I clusters is uncomfortably low ($\sim 0.55 M_\odot$; Cox, Hodson and Clancy 1983). The Cox, Hodson and Clancy paper was notable for having greatly expanded the class of double-mode RR Lyrae stars, now called RRd stars. Later work by Simon and Cox (1991) confirmed the earlier mass estimates.

6. Other Stars and the Sun

Art has collaborated with Starrfield and Hodson on several papers studying pulsating white dwarfs. Cox, Hodson and Starrfield (1980) examined H and He II ionization driving as the cause of ZZ Ceti pulsation. Starrfield, Cox and Hodson (1980) discovered the pulsating PG 1159 variables, hot white dwarfs possibly driven by the C and O K-ionization zones. Further studies of the instability mechanisms of the PG 1159 stars were made by Starrfield, Cox, Hodson and Pesnell (1983) and Starrfield, Cox, Kidman and Pesnell (1984). Additional work on pulsating DA white dwarfs was done by Starrfield, Cox, Hodson and Clancy (1983) and Cox, Starrfield, Kidman and Pesnell (1987).

Solar oscillation work first appears in Art's bibliography in 1985, when he joined Kidman and Newman in a paper on solar oscillation constraints on mixing (Cox, Kidman and Newman 1985). Turbulent diffusivity could reduce the neutrino flux, but only by violating the oscillation data. Cox, Guzik and Kidman (1989) made a very careful study of solar evolution including the diffusive processes of settling, thermal diffusion and concentration diffusion, in order to find the effect these processes have on the neutrino flux and the oscillation data. The results that were found indicated an aggravated solar neutrino problem. Cox, Guzik and Raby (1990) investigated a pair of more speculative solar models; the cosmion (WIMP) model seemed slightly promising. Art reviewed some of this work in 1989 (Cox 1990). Guzik and Cox (1991,1992,1993a) pursued the work by improving the equation of state description and the opacity (using Mihalas, Hummer and Däppen and OPAL, respectively) in the stellar evolution code, and by fine tuning the convection zone helium abundance to account for helium diffusion. The differences between the solar oscillation data and the best of these "standard" models are quite small indeed.

7. Opacity Effects on Pulsation and the Beta Cephei Stars

In seeking to reconcile the pulsational and evolutionary masses of cepheids, Art, with Hodson and King, settled on the idea of a surface layer enhanced in helium as a plausible explanation. The effect that was needed in the envelope structure was a flatter density gradient, and an outward gradient of the mean atomic weight, μ, produced that. Simon (1981) explored other ideas for flattening the density profile, including an increase of the opacity. A higher opacity means that the envelope temperature is generally increased, which makes the pressure scale height larger, and therefore the density gradient is reduced. In a subsequent paper, Simon (1982) suggested an opacity increase of 2–3 times at temperatures above 10^5 K. The title of this paper was "A Plea for Reexamining Heavy Element Opacities in Stars." His plea was heard by the Opacity Project (Seaton, Mihalas, Pradhan, *et al*) as well as by the OPAL team (Rogers and Iglesias). OPAL was first with a response (Iglesias, Rogers and Wilson 1987, 1990; Iglesias and Rogers 1991a,b). The OP results, which turned out to be quite similar, appeared later (Seaton, *et al,* 1994).

The OP and OPAL efforts produced opacities for cepheids that were substantially higher than Cox&Tabor or the Los Alamos Opacity Library at temperatures above 10^5 K, just as Simon asked. The first verification that the new opacities did the job for the double-mode cepheids was by Moskalik, Buchler and Marom (1992). Their Petersen diagram is shown in Figure 7.

The confirmation of the opacity explanation of the cepheid mass problem is a victory, of course, for Simon, and for all the opacity theorists; but it is also a triumph for Art, who would not let the problem go away, and who has always supported using the best opacity data available. Art had already published his application of OPAL

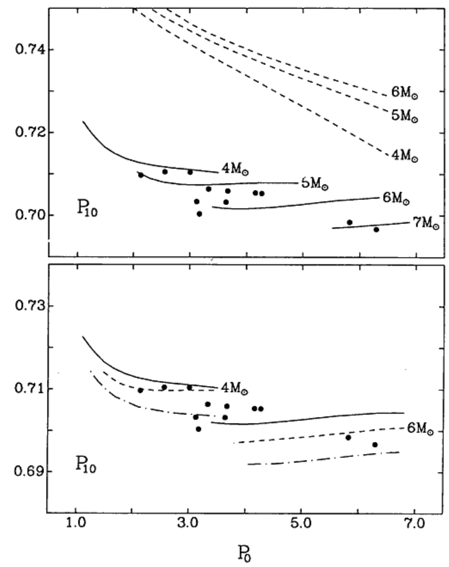

Figure 7. The Petersen diagram for galactic double-mode cepheids. Abscissa: fundamental period (days); ordinate: 1st overtone period/fundamental period. Dots: observed cepheids. Upper panel: solid curves using OPAL, Dashed curves using Los Alamos Opacity Library; both with Becker-Iben-Tuggle (1977) $Z = 0.02$ mass-luminosity relation. Lower panel: all with OPAL, three choices of mass-luminosity relation. (From *The Astrophysical Journal,* by permission.)

opacities to the RRd stars (Cox 1991), in which he showed that the inferred mass of the double-mode RR Lyrae stars should be increased by $0.1\,M_{\odot}$, and that the mass difference between Oosterhoff I and Oosterhoff II cluster RRd stars could vanish.

Another triumph of the new opacities, also predicted by Simon (1982), was the destabilization of the beta cephei stars. The early work by Stellingwerf (1979) had shown that the beta cephei models were *almost* destabilized by a feature, the "Stellingwerf bump," in the opacity near 1.5×10^5 K, where the Planck maximum coincides with the He II K edge. The Stellingwerf bump in the Cox&Tabor opacities was too mild to produce instability, but by coincidence, the bump in the new opacities, which is due mostly to Fe lines, falls in the same place and is much more pronounced. Cox, Morgan, Rogers and Iglesias (1992) and Moskalik and Dziembowski (1992) harvested this result. The mechanism of the beta cephei stars had been a long-standing puzzle, and it was satisfying indeed to see this one solved at the same stroke as the other puzzle: the mass problem.

8. Recent Work

Art and our conference chairman, Joyce Guzik, have continued with the analysis of solar oscillations. The solar seismology results show a sound speed discrepancy near the base of the convection zone, and Guzik and Cox (1993a) tried including composition diffusion as well as variation in the convection zone depth to explain it. They returned to the topic in Guzik and Cox (1995), with the additional ingredient of enhanced early mass loss. A review appeared in Cox and Guzik (1995). Art contributed to the extensive review of solar seismology in *Science* last year (Christensen-Dalsgaard, *et al*, 1996).

Art and Ostlie returned to the Long Period Variables in a study (Cox and Ostlie 1993) of linear and non-linear LPV models aimed at resolving the problem of the actual mode of pulsation. This is one area where further work will be needed.

The RR Lyrae stars continue to get some attention: Guzik and Cox (1993b) show a systematic study of RR Lyrae pulsation using OPAL opacities. They repeat the conclusion of Cox (1991) that the RR Lyrae masses may be increased. (For another view see Icko Iben's contribution to this workshop.) Art reviewed the mass question and the direction of evolution in Cox (1995).

Finally, in the most recent work Art has turned to Luminous Blue Variables (Cox, Guzik, Soukup and Morgan 1995). The LBVs may actually lie in the luminous extension of the beta cephei instability strip, as pointed out by Moskalik and Dziembowski (1992). The instability results of Cox, *et al*, feature prominently the "strange" mode that we will hear more of this week. And on Wednesday Art, with Guzik, Soukup and Despain, will present some non-linear models of LBVs that show a violent strange-mode instability combined with a relaxation oscillation back and forth across the Eddington limit.

Acknowledgments. I primarily want to acknowledge Art himself. Art is the godfather of pulsation theory, which has been perhaps the most successful of all areas of theoretical astrophysics. His extraordinary energy has inspired all of us in this room. His persistent attack on the basic problems, and the vigor with which he has kept the attention of the community focussed on them—through meetings like this one—are significant ingredients in that success.

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract W-7405-Eng-48.

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